Session 2 - Summary of definitions

AGM Belief Revision Postulates

Basic postulates:

$$B * \phi \text{ is a belief set} \qquad \text{(Closure)}$$

$$\phi \in B * \phi \qquad \text{(Success)}$$

$$\text{If } \phi \nvdash \bot, \text{ then } Cn(B * \phi) \neq \mathcal{L} \qquad \text{(Consistency)}$$

$$B * \phi \subseteq Cn(B \cup \{\phi\}) \qquad \text{(Inclusion)}$$

$$\text{If } B \nvdash \neg\phi, \text{ then } B * \phi = Cn(B \cup \{\phi\}) \qquad \text{(Vacuity)}$$

$$\text{If } Cn(\{\phi\}) = Cn(\{\psi\}), \text{ then } B * \phi = B * \psi \qquad \text{(Congruence)}$$

Additional postulates:

$$B * (\phi \wedge \psi) \subseteq (B * \phi) + \psi \qquad \text{(Superexpansion)}$$

$$\text{If } B * \phi \nvdash \neg\psi, \text{ then } (B * \phi) + \psi \subseteq B * (\phi \wedge \psi) \qquad \text{(Subexpansion)}$$

Truth-Sets

> **Definition 1** (Truth-Set).
>
> Let $\phi \in \mathcal{L}$. The truth-set function $[\cdot] : \mathcal{L} \to \mathcal{P}(W)$ maps a sentence to the set of its models:
>
> $$[\phi] := \{w \in W : w \models \phi\}$$
>
> We can extend the domain of $[\cdot]$ to include *sets* of sentences. Let $\Gamma \subseteq \mathcal{L}$ be a set of sentences. Its truth-set is the intersection of the truth-sets of all its members:
>
> $$[\Gamma] := \bigcap_{\phi \in \Gamma} [\phi] = \{w \in W : w \models \phi \text{ for all } \phi \in \Gamma\}$$

> **Lemma 2.**
>
> *Let $\phi, \psi \in \mathcal{L}$ and let $\Gamma, \Delta \in \mathcal{P}(\mathcal{L})$. The semantic sets behave according to the following logical rules:*
>
> $$\begin{array}{ll} 1. & \phi \vdash \psi \iff [\phi] \subseteq [\psi] \\ 2. & \Gamma \subseteq \Delta \implies [\Delta] \subseteq [\Gamma] \\ 3. & \Gamma \subseteq \Delta \iff [\Delta] \subseteq [\Gamma] \quad \text{(if } \Delta \text{ is a belief set)} \end{array}$$

Properties of Theory Function and Truth-Set Function

Theory Function $T(\cdot)$: For all $V \subseteq W$:

$$T(V) := \{\phi \in \mathcal{L} : V \subseteq [\phi]\}$$

> **Lemma 4.**
>
> *Let $V \subseteq W$ be any set of worlds, and $\Gamma \subseteq \mathcal{L}$ be any set of sentences.*
>
> 1. *$T(V)$ is a belief set, i.e. $T(V) = Cn(T(V))$.*
> 2. *$T([\Gamma]) = Cn(\Gamma)$. Consequently, $T([\Gamma]) = \Gamma \iff \Gamma$ is a belief set.*
> 3. *$V \subseteq [T(V)]$. Furthermore, the equality $[T(V)] = V$ holds for all $V \subseteq W$ if, and only if, Φ is finite.*

Properties of $\preceq_B$:

1. **Connectedness**: For any worlds $w_1, w_2 \in W$: Either $w_1 \preceq_B w_2$ or $w_2 \preceq_B w_1$.
2. **Transitivity**: For any worlds $w_1, w_2, w_3 \in W$: If $w_1 \preceq_B w_2$ and $w_2 \preceq_B w_3$, then $w_1 \preceq_B w_3$.
3. **Centeredness**:
    1. If $w_1, w_2 \in [B]$, then $w_1 \preceq_B w_2$.
    2. If $w_1 \in [B]$ and $w_2 \notin [B]$, then $w_1 \prec_B w_2$.
4. **Limit Assumption**: For any sentence $\phi \in \mathcal{L}$: If $[\phi] \neq \emptyset$, then there exists at least one world $w_1 \in [\phi]$ such that $w_1 \preceq_B w_2$ for all $w_2 \in [\phi]$.

Definition of $B * \phi$:

$$B * \phi := T(\min_B([\phi]))$$

where $\min_B([\phi]) := \{w \in [\phi] : \text{For all } w' \in [\phi], w \preceq_B w'\}$.